

Log Properties

$$1) \log_b(xy) = \log_b x + \log_b y$$

$$2) \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$* 3) \log_b(x^r) = r \log_b x$$

$$4) \log_b b = 1; \log_b 1 = 0$$

$$5) \log_b x = \frac{\log_c x}{\log_c b}$$

$$6) b^{\log_b x} = x$$

Example

$$\textcircled{10} \quad 5.3(10^x) = 2$$

$$\frac{5.3(10^x)}{5.3} = \frac{2}{5.3}$$

$$10^x = 0.3774$$

$$\log_{10} x = \log 0.3774$$

$$x \cdot \log_{10} 10 = \log(0.3774)$$

$$x = \log(0.3774)$$

$$x = -0.423198$$

$$3^{x+2} = 27^{x-1} \cdot 9 \quad \text{"like bases"}$$

$$3^{x+2} = 3^{3(x-1)} \cdot 3^2$$

$$3^{x+2} = 3^{3x-3} \cdot 3^2$$

$$3^{x+2} = 3^{3x-1}$$

$$x+2 = 3x-1$$

$$\begin{array}{r} -x \qquad -x \\ 2 = 2x - 1 \\ +1 \qquad +1 \end{array}$$

$$3 = 2x$$

$$\frac{3}{2} = x$$

"unlike bases"

$$2^{3x-1} = 5^{x+4}$$

$$\ln 2^{3x-1} = \ln 5^{x+4}$$

$$(3x-1) \cdot \ln 2 = (x+4) \cdot \ln 5$$

$$(3 \cdot \ln 2)x - \ln 2 = x \ln 5 + 4 \ln 5$$

$$(3 \cdot \ln 2)x - x \ln 5 = 4 \ln 5 + \ln 2$$

$$x(3 \ln 2 - \ln 5) = 4 \ln 5 + \ln 2$$

$$x = \frac{4 \ln 5 + \ln 2}{3 \ln 2 - \ln 5}$$

$$(3x-1) \cdot \underbrace{\ln 2}_{0.6931} = (x+4) \underbrace{\ln 5}_{1.609}$$

"Equations w/ Logs"

$$\log_3(x-1) + \log_3 2 = 1$$

$$\log_3 2(x-1) = 1$$

$$2x-1 = 3$$

+2 +2

$$x = \frac{5}{2}$$

Exercises

34 35 compound Interest Formula

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

A = accumulated amt

r = rate

t = time

n = # compounded

P = principal/initial amt

Compounded Continuously

$$A = Pe^{rt}$$

$P = 1000$, triples - 3000

continuously... $A = Pe^{rt}$

$$\frac{3000}{1000} = \frac{1000e^{0.10t}}{1000}$$

$$3 = e^{0.10t}$$

$$\ln 3 = \ln e^{0.10t}$$

$$\ln 3 = 0.10t \cdot \frac{\ln e}{\log_e e}$$

$$\frac{\ln 3}{0.10} = t$$

$$10.9861 = t$$

roughly

11 years

Exercises

(54) half life = time for half of a substance to decay

$$Q = Q_0 e^{kt} \quad k > 0 \text{ "growth"}$$

$$k < 0 \text{ "decay"}$$

Strontium 90 $\frac{1}{2}$ life = 28 years

$$Q_0 = A = \text{starting amt}$$

$$Q_0 = 1000$$

in 28 years, $\frac{1}{2}$ of it is left

$$Q = 500 \quad (28, 500)$$

$$Q = Q_0 e^{kt} \quad \text{"Equation set-up"}$$

$$500 = 1000 e^{k(28)}$$

$$\frac{1}{2} = e^{28k}$$

$$\ln(0.5) = \ln e^{28k}$$

$$\frac{\ln(0.5)}{28} = k = -0.02476$$

$$Q = Q_0 e^{-0.02476t}$$

$$b) Q_0 = 5000$$

$$Q = 2000$$

$$\text{so, } 2000 = 5000 e^{-0.02476t}$$

$$0.4 = e^{-0.02476t}$$

$$\ln(0.4) = \ln e$$

$$\ln(0.4) = -0.02476t \cdot \ln e$$

$$\frac{\ln(0.4)}{-0.02476} = t$$

$$37 \text{ yrs} = t$$